# Reliability Modeling of Wind Farms

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## ABSTRACT

Wind generation is one of the most useful sources of renewable energy for the production of electrical energy. The technical characteristics of wind generation make existing conventional generation models not directly applicable. This paper presents an analytical method for the reliability assessment of the flat-rated wind turbine generation systems. An example of flat rating is that of the MOD-2, a second-generation class of wind turbines. The power-velocity characteristic of the flat-rated wind turbines is employed in this paper to model the operating behavior of the installed wind turbine generators. For wind-power potential estimation, the Weibull distribution model is used. The performance of the proposed model is demonstrated with case studies.

**Keywords:** Reliability evaluation, Weibull probability distribution function, wind turbine generation

## I. NOMENCLATURE

| | |
|---|---|
| $\lambda$ | Failure rate in failure/year. |
| $\mu$ | Repair rate in repair/year. |
| $FOR$ | Forced Outage Rate. |
| $MTTF$ | Mean Time To Failure. |
| $MTTR$ | Mean Time To Repair. |
| $MTBF$ | Mean Time Between Failures. |
| $P_{total}$ | Total power of wind stream. |
| $\dot{m}$ | Mass flow rate of wind stream. |
| $V_i$ | Velocity of wind stream. |
| $\rho$ | Air mass density. |
| $A$ | Area of wind stream. |
| $P_{\max}$ | Maximum power. |
| $\eta$ | Efficiency. |
| $\eta_{\max}$ | Maximum efficiency. |
| $V_{Cut-in}$ | Cut-in velocity. |
| $V_{Cut-out}$ | Cut-out velocity. |
| $V_{Rating}$ | Rated velocity. |
| $\overline{V}$ | Mean wind velocity. |
| $\overline{V}_E$ | Mean energy velocity. |
| $n$ | Number of wind velocity observations. |
| $V$ | Wind velocity. |
| $V_r$ | Reference velocity. |
| $H_r$ | Reference height. |
| $P(X)$ | Probability of X. |
| $\Gamma(X)$ | Gamma function of X. |
| $DG$ | Dispersed Generation. |
| $WTG$ | Wind Turbine Generation |

## II. INTRODUCTION

Public environmental concerns with electrical energy derived from fossil resources have created an increased interest in the development and use of alternative sources. The technologies for the generation of electrical energy from renewable energy sources have evolved in recent years from small experimental plants to a real option for the utilities to produce significant amounts of electric power. Wind generation is one of the most successful sources of renewable energy for the production of electrical energy. Many utilities throughout the world are considering using wind energy as a substitute for conventional generation due to its huge potential and minimal pollution [1]-[2].

One of the most promising applications of wind energy is in remote, windy places which have weak and autonomous power systems. There are several Iranian regions meeting these conditions. As the number of wind power installations connected to existing electric power networks grows rapidly worldwide, there is a need to study these energy sources and assess their effects on the system. To evaluate the connection of wind energy sources to the electric supply system and particularly to distribution networks, it is necessary to evaluate how the reliability and production of the system may be affected. However, the technical characteristics of wind generation make existing conventional generation models not directly applicable [2]-[3].

From a reliability modeling and evaluation viewpoint, dispersed generation can generally be grouped into two main types: those which have an output dependent on a variable energy source (e.g. wind, solar) and cannot be prescheduled and those that are not so dependent (e.g. hydro, gas, and diesel) and could be prescheduled. The latter type can be modeled using conventional generation approaches and their contribution to the system supply is only dependent on need and the availability of the units themselves. The former, however, are much more difficult to deal with because their contribution to the system supply is also dependent on the primary source of energy being available as well as need and unit availability [4]. Several excellent textbooks and references are available that provide a detailed description of the modeling and evaluation efforts [1]-[15]. This paper will not attempt to focus on these references, but will use their approach and philosophy to develop a new method.

The paper has the following structure. A brief introduction to reliability evaluation of generation systems is provided in section III; section IV describes concepts and characteristics of wind turbine generation; numerical examples are presented in section V; and finally, conclusions are presented in section VI.

## III. RELIABILITY EVALUATION OF ELECTRICITY GENERATING SYSTEMS

The determination of the required amount of system generating capacity to ensure an adequate supply is an important aspect of electric power system planning and operation [16]. The basic generating unit parameter used in static capacity evaluation is the probability of finding the unit on forced outage at some distant time in the future. This probability is defined as the unit unavailability, and historically in power system applications it is known as the unit forced outage rate (FOR),

$$\text{Unavailability}\,(FOR) = \frac{\lambda}{\lambda + \mu} \qquad (1)$$

$$\text{Availability} = \frac{\mu}{\lambda + \mu} \qquad (2)$$

The concepts of availability and unavailability as illustrated in equations (1) and (2) are associated with the simple two-state model shown in Fig. 1. This model is directly applicable to a generating unit which is either operating or forced out of service.

The two-state model of Fig. 1 is the situation in which $\lambda$ and $\mu$ are constant values. In this case

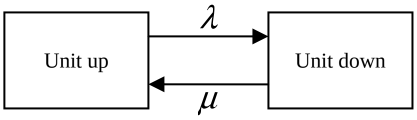

**Fig. 1- Two-state Model for a Generating Unit**

$$\text{Mean time to failure} = MTTF = {}^{1}\!/_{\lambda} \qquad (3)$$

$$\text{Mean time to repair} = MTTR = {}^{1}\!/_{\mu} \qquad (4)$$

$$\text{Mean time between failures} = MTBF = MTTF + MTTR \qquad (5)$$

This can be illustrated using the representation shown in Fig. 2.

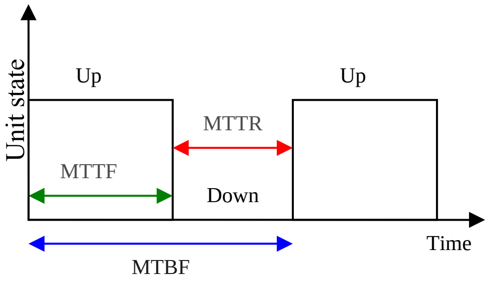

**Fig. 2- Average History of a Generating Unit.**

**Table1 – Capacity Outage Probability Table for a Generation System With Two Non-Identical Units**

| State | Unit 1 | Unit 2 | Capacity in | Capacity out | State probability |
|---|---|---|---|---|---|
| 1 | Up | Up | $C_1 + C_2$ | 0 | $(1 - FOR_1) \cdot (1 - FOR_2)$ |
| 2 | Up | Down | $C_1$ | $C_2$ | $(1 - FOR_1) \cdot FOR_2$ |
| 3 | Down | Up | $C_2$ | $C_1$ | $FOR_1 \cdot (1 - FOR_2)$ |
| 4 | Down | Down | 0 | $C_1 + C_2$ | $FOR_1 \cdot FOR_2$ |

The generation model is sometimes known as a capacity outage probability table. As the name suggests, it is a simple array of capacity levels and the associated probabilities of existence. If all the units in the system are identical, the capacity outage probability table can be easily obtained using the binomial distribution. Otherwise, the units can be combined using basic probability concepts and this approach can be extended to a simple but powerful recursive technique in which units are added sequentially to produce the final model. (Table 1)

## IV. WIND TURBINE GENERATION

### *Wind Power*

The total power of a wind stream is equal to the rate of the incoming kinetic energy of that stream, or

$$P_{total} = \frac{1}{2}\dot{m}V_i^2 \qquad (6)$$

The mass flow rate is given by the continuity equation

$$\dot{m} = \rho A V_i \qquad (7)$$

Thus

$$P_{total} = \frac{1}{2}\rho A V_i^3 \qquad (8)$$

Thus the total power of a wind stream is directly proportional to its density, area, and the cube of its velocity. It can be proved that the total power cannot all be converted to mechanical power. There is a maximum power $P_{\max}$ which can be obtained from the wind

$$P_{\max} = \frac{8}{27}\rho A V_i^3 \qquad (9)$$

The ideal, or maximum, theoretical efficiency $\eta_{\max}$ (also called the power coefficient) of a wind turbine is the ratio of the maximum power obtained from the wind to the total power of the wind, or

$$\eta_{\max} = \frac{P_{\max}}{P_{total}} = 0.5926 \qquad (10)$$

In other words, a wind turbine is capable of converting no more than 60 percent of the total power of a wind to useful power.

Since a wind-turbine wheel cannot be completely closed, and because of spillage and other effects, practical turbines achieve some 50 to 70 percent of the ideal efficiency. The real efficiency $\eta$ is the ratio of actual to total power

$$P=\eta P_{\max} = \frac{1}{2}\eta\rho A V_i^3 \qquad (11)$$

Where $\eta$ varies between 30 and 40 percent for real turbines.

## Flat-rated Wind Turbine Generation

Severe fluctuations in power are undesirable. They pose power-oscillation problems on the grid and severe strains on the wind turbine hardware. It is, therefore, more cost-effective to design a wind turbine to produce rated power at less than the maximum prevailing wind velocity, and to maintain a constant output at all wind speeds above rating. This is called flat rating (Fig. 3).

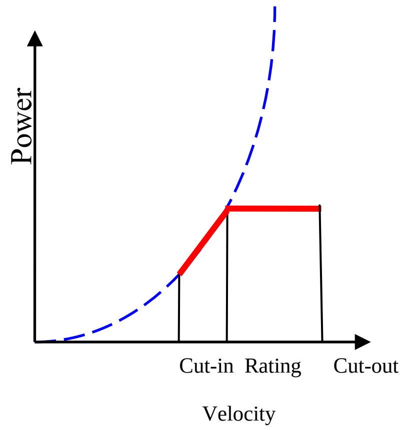

**Fig. 3- Power-Velocity Characteristics of a Flat-Rated Wind Turbine.**

Because of a severe loss in efficiency and power at low wind velocities, a wind turbine is designed to come into operation at a minimum wind speed, called the cut-in velocity. To protect the turbine wheel against damage at very high wind velocities, it is designed to stop operation at a cut-out velocity. Thus the wind turbine operates with variable load over a narrow range between the cut-in and the rated velocities and at constant power between the rated and cut-out velocities and ceases operation above the cut-out velocity [17].

## Wind Speed Model

Wind-power potential can be estimated from the mean wind velocity $\overline{V}$, which is based on measurements over a period of time. It is given by:

$$\bar{V}=\frac{\sum_{i=1}^{n} V_i}{n} \qquad (12)$$

where the numerator is the sum of all velocity observations and $n$ is the number of these observations.

Power plant sizing would be underestimated if it were rated at the mean wind velocity. A more representative figure would be based on the mean energy velocity $\bar{V}_E$, which because of the power dependence on the cube of the wind velocity, is given by

$$\bar{V}_E=\sqrt[3]{\frac{\sum_{i=1}^{n} V_i^3}{n}} \qquad (13)$$

For estimation purposes, the Weibull distribution model has been found useful and appropriate for wind-turbine performance analysis by many investigators [17]. This model gives the probability that the wind velocity is greater than a selected value $V$ for a locality where the mean wind velocity $\bar{V}$ is known. It is given at reference height $H_r = 9.1$ m, where $V_i$ and $V$ are in m/s, by

$$P(V_i > V)= e^{-(V/C_r)^{K_r}} \qquad (14)$$

where

$$K_r = 1.09 + 0.2V \qquad (15)$$

$$C_r = \frac{\bar{V}}{\Gamma(1+\frac{1}{K_r})} \qquad (16)$$

$\Gamma =$ gamma function

A height above ground of 9.1 m is usually used as a reference elevation for which wind velocities are listed. The velocity to be used in determining the power of a wind turbine, however, is that at the hub of the turbine wheel. The ratio of wind velocity $V$ at a height $H$ to the reference velocity $V_r$ at reference height $H_r$ is given by a wind-shear model as

$$\frac{V}{V_r}=\left(\frac{H}{H_r}\right)^{\alpha} \qquad (17)$$

where

$$\alpha = \alpha_0 \left( 1 - \frac{\log V_r}{\log V_0} \right) \qquad (18)$$

At elevations H, for values other than the reference $H_r = 9.1$ m, the Weibull distribution model is used, but the parameters $K_r$ *and* $C_r$ are modified to $K_H$ *and* $C_H$, given by

$$K_H = \frac{K_r}{1 - \alpha_0 \left[ \left( \log\left( \frac{H}{H_r} \right) \right) \Big/ \log V_0 \right]} \qquad (19)$$

and

$$C_H = C_r \left( \frac{H}{H_r} \right)^{\alpha_H} \qquad (20)$$

where

$$\alpha_H = \alpha_0 \left( 1 - \frac{\log C_r}{\log V_0} \right) \qquad (21)$$

$$\alpha_0 = \left( \frac{Z_0}{H_r} \right)^{0.2} \qquad (22)$$

$Z_0$ =0.4m and $V_0$ =67.1m/s

### *WTG Modeling for Reliability Evaluation*

As described in previous sections, the output of a wind turbine generation unit is a function of the wind speed. When a WTG is in the normal operating condition, it can be represented by a three-state model. Up1, Up2 and Down are three states, which represent variable, constant and zero outputs, respectively, in terms of wind speeds. A WTG can also suffer a forced outage, which can be represented by Up and Down states. In order to consider the joint effects of both wind speed and the forced outage, a WTG can be represented by the three-state model shown in Fig. 4 and table 2. A wind farm usually consists of many units and the specified wind velocity is assumed to be the same for all the units in the farm. The power output of a wind farm is the summation of the output of all the available units [7].

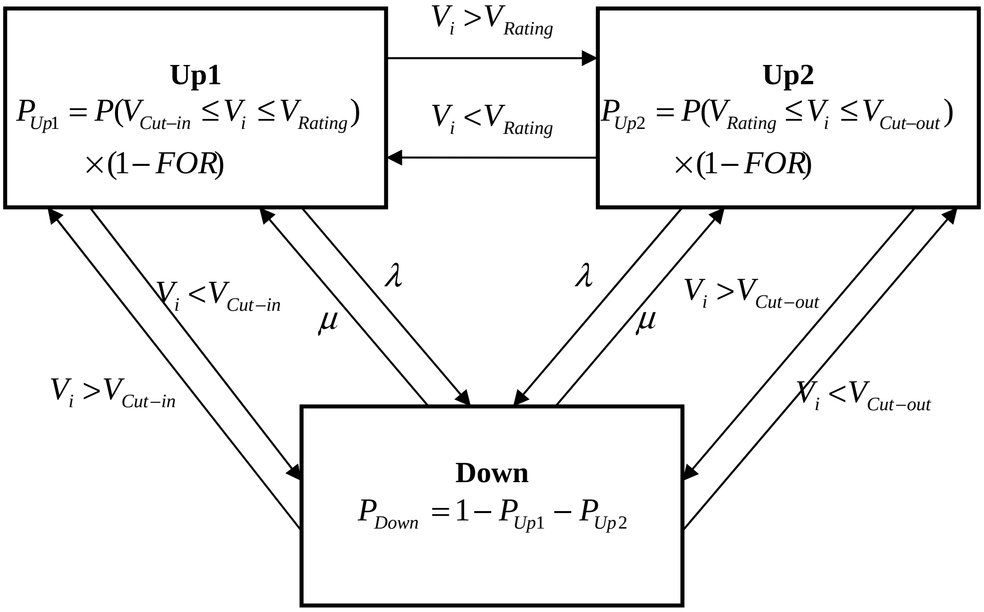

**Fig. 4- Three-State Markov Model of a Flat-Rated Wind Turbine Generation System**

The total output of a generation station is obtained by appropriately aggregating the outputs of each individual generator. The conventional approach to aggregate these outputs using probabilistic analytical techniques is the convolution of their individual capacity outage probability tables [17]. This approach is applicable to non-intermittent sources such as gas and diesel, but is not valid for wind generators. The problem is that the outputs of units such as wind turbines are dependent on a common source, e.g. the wind. Therefore there is dependence between the different generation output states, whereas independence is an underlying assumption in the convolution of capacity outage probability tables. Consequently it is not possible to calculate a generation output capacity table by convolving the output table of each wind turbine. Determination of the output characteristics of a wind farm for reliability analysis requires the simultaneous consideration of all wind turbines [4].

**Table 2- Flat-rated Wind Turbine Generation System –Three State Representation**

| Unit state | State probability | Expected output power |
|---|---|---|
| Up1 | $(1-FOR)P(V_{Cut-in} < V_i < V_{Rating})$ | $\frac{1}{2}\eta\rho A \frac{\int_{V_{Cut-in}}^{V_{Rating}} \left( V^3 e^{-\left(V/C_H\right)^{K_H}} \right)\cdot dV}{\int_{V_{Cut-in}}^{V_{Rating}} \left( e^{-\left(V/C_H\right)^{K_H}} \right)\cdot dV}$ |
| Up2 | $(1-FOR)P(V_{Rating} < V_i < V_{Cut-out})$ | $\frac{1}{2}\eta\rho A V_{Rating}^3$ |
| Down | $1-(1-FOR)P(V_{Cut-in} < V_i < V_{Cut-out})$ | 0 |

## V. NUMERICAL EXAMPLES

### *Reliability of Wind Turbine Unit*

Consider a flat-rated wind turbine generation unit, with an outage rate of 3%. The wind pattern that follows the Weibull distribution model has a reference mean velocity 10m/s, measured at the reference height above ground of 9.1m at 1 standard atmosphere pressure and $20^{\circ}$C temperature. Consider a turbine with a hub 50m above ground that has cut-in, rated, and cut-out velocities of 4.97, 8.84, and 16.29m/s, respectively, all at the reference height above ground of 9.1m. The turbine wheel has a 1960m$^2$ cross-sectional area (50m diameter). The wind turbine and generator efficiencies are 38% and 94%, respectively.

Using (14) to (22), the velocities at an altitude of 50m above ground and their probabilities are computed as:

$$\overline{V} = 15.11\text{m/s}$$

$$V_{Cut-in} = 7.51\text{m/s}$$

$$V_{Rating} = 13.36\text{m/s}$$

$$V_{Cut-out} = 24.61\text{m/s}$$

$$P\left(V > V_{Cut-in}\right) = 0.975366$$

$$P\left(V > V_{Rating}\right) = 0.923235$$

$$P\left(V > V_{Cut-out}\right) = 0.104641$$

The results are shown in Table 3.

**Table 3- Three State Capacity Outage Probability Table of WTG Unit**

| Unit state | State probability | Expected output power (Watt) |
|---|---|---|
| Up1 | 0.050567 | 435442 |
| Up2 | 0.794036 | 854785 |
| Down | 0.155397 | 0 |

## *Wind Farm Model*

Consider a wind farm consisting of the same WTG units as the unit modeled in Table 3. Without the loss of generality, it is assumed that there are only two units in the farm. This assumption can easily be removed allowing more units to be considered.

The state probability analysis of the wind farm is shown in Table 4. As discussed in previous sections, the outputs of units are dependent on the wind. Consequently it is not possible to calculate the generation output capacity table by convolving the output table of each unit.

**Table 4- Example State Probability Analysis of Wind Farm**

| State | Wind velocity | Unit 1 | Unit 2 | State probability | Expected output (Watt) |
|---|---|---|---|---|---|
| 1 | $V_{Cut-in}<V<V_{Rating}$ | Up | Up | 0.047578 | 870884 |
| 2 | | Up | Down | 0.001471 | 435442 |
| 3 | | Down | Up | 0.001471 | 435442 |
| 4 | | Down | Down | 0.000046 | 0 |
| 5 | $V_{Rating}<V<V_{Cut-out}$ | Up | Up | 0.747108 | 1709570 |
| 6 | | Up | Down | 0.023106 | 854785 |
| 7 | | Down | Up | 0.023106 | 854785 |
| 8 | | Down | Down | 0.000715 | 0 |
| 9 | $V<V_{Cut-in}$ or $V>V_{Cut-out}$ | Up | Up | 0.146213 | 0 |
| 10 | | Up | Down | 0.004522 | 0 |
| 11 | | Down | Up | 0.004522 | 0 |
| 12 | | Down | Down | 0.000140 | 0 |

Sometimes it is not valid to assume all the WTG at a wind farm see the same wind velocity. Determination of location for a wind farm is usually based upon assessment of the wind patterns. Layout of the wind farm is a challenging task for which no completely comprehensive method has been developed. In fact, extensive computer simulation and innovative thinking are often required to balance the myriad objectives involved: optimum recovery of wind energy, lack of interference of one turbine with another and operating cost (O&M access). Very often, the best locations for wind farms are on barren hillsides and similar locations where only one or perhaps two wind turbines can be located in series along the wind’s direction.

## VI. CONCLUSION

An analytical probabilistic model for the evaluation of the reliability of wind energy systems has been presented. The model considers the stochastic nature of wind, the failure and repair rates of the WTG units, and the flat rated wind turbine output characteristics in detail. The model can be easily integrated into existing reliability evaluation models for electric power networks. The performance of the developed model has been demonstrated with computational results using a flat-rated test system. Simulation results for two case studies are presented to show the different aspects of reliability evaluation of wind farms. The results show the effectiveness of the proposed method for wind farm modeling.

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